Vectors

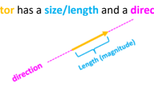
Vector Basics (too easy to come up, but need to know as a prerequisite)

Vector Notation (3 Ways)

Vectors have a size and a direction

This is a vector (a line with an arrow)

A vector has a size/length and a direction

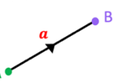
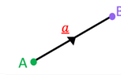


size = length = magnitude
a direction (certain path)

Think of a gun. A bullet fired has a size and a certain direction that it travels

The diagram below shows a vector representing the move from point A to point B with an arrow going from A to B

Vector Notation (there are 3 ways to write a vector)

Way 1: Using an arrow on top of two capital letters.	Way 2: Using bold lowercase letters $\boldsymbol{a}$	Way 3: Underlining lowercase letters $\underline{a}$
For example, $\overrightarrow{AB}$	For example, $\boldsymbol{a}$	For example, $\underline{a}$
$\overrightarrow{AB}$ means start at A and end at B		
The first letter tells us where we start, and the second letter tells us where we end		

You may be wondering why are there 2 lowercase notations in Ways 2 and 3 (boldface and underlined)? This is mostly because of how it's written. Generally, computer printing uses boldface and human handwriting is underlined because it is easier to underline than to try and make bold by hand. The direction of the arrow matters. If we go in the **opposite direction** to the arrow then the vector is **negative.**

$\overrightarrow{AB} = \boldsymbol{a}$ or $\underline{a}$

$\overrightarrow{BA} = -\boldsymbol{a}$ or $-\underline{a}$

Note: if you just see the vector written as $\boldsymbol{a}$ or $\underline{a}$, it means $\overrightarrow{OA}$ (comes from origin O to A and it called a position vector)

A position vector is used to tell how far and in which *direction a point is with respect to some arbitrarily chosen origin*. You will see this in more detail in the position vector section.

Direction Explained In More Detail

We call the **start** place the **tail** and the **end** place the **head.**

We have to respect the direction that the vector displays. The direction of the arrow indicates a **positive**. We can't just change/swap the direction of the arrow. This means if we go in the reverse the direction of the arrow we must write a negative.

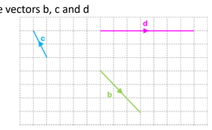
$\overrightarrow{BA} = -\overrightarrow{AB}$

Otherwise, if we don't distinguish the difference with vectors, it is as bad as saying 2 is the same as -2 when it is **clearly not!**

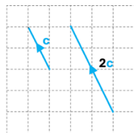
Visual Representation Of Vectors - Adding/Subtracting/Multiplying Vectors

Multiplying a Vector by a number

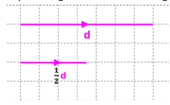
Let's say we have some vectors b, c and d



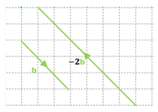
If we know $\boldsymbol{c}$ we can draw $2\boldsymbol{c}$ by drawing a vector double the length in the same direction



If we know $\boldsymbol{d}$ we can draw $\frac{1}{2}\boldsymbol{d}$ by drawing a vector half the length in the same direction

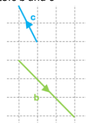


If we know $\boldsymbol{b}$ we can draw $-2\boldsymbol{b}$ by drawing a vector half the length in the OPPOSITE direction since the negative



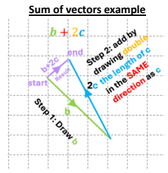
Adding/Subtracting Vectors

Let's say we have some vectors b and c



If we know $\boldsymbol{b}$ and $\boldsymbol{c}$ we can work out sums and differences of these vectors

Sum of vectors example



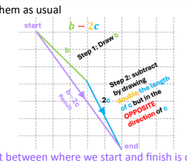
The result between where we start and finish is our answer

Note: It doesn't matter which order we add them, we get the same result (we could have stared with 2c and added b after)

Example of subtracting vectors

We can also subtract one vector from another

- first we reverse the direction of the vector we want to subtract
- then add them as usual

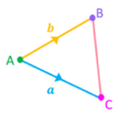


The result between where we start and finish is our answer

The order we do this matters $\boldsymbol{b} - 2\boldsymbol{c} \neq 2\boldsymbol{c} - \boldsymbol{b}$

Vector Diagrams

A typical vector diagram looks like the following with big letters (capital letters) at the points and little letters (lowercase letters) between the points.



Key:
The routes in between the big letters can be represented by little letters

$\overrightarrow{AB} = \boldsymbol{b}$

$\overrightarrow{AC} = \boldsymbol{a}$

Let's say we want to find the vector $\overrightarrow{BC}$. Recall that the first letter B tells us where we start and the 2nd letter C tells us where we end. This means we start at B and end at C.

Yes, we have a direct route from B to C (the pink route). However, we don't know this route which means we have to use the indirect route in order to find $\overrightarrow{BC}$

$\overrightarrow{BC}$

$= B$ to A to C

This is the same as

B to A and then A to C

$= \overrightarrow{BA} + \overrightarrow{AC}$

We are not done though. We always need to always give our answers in terms of **lowercase letters**, **not uppercase letters**! We now replace with the little letters that represent each.

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\boldsymbol{b} + \boldsymbol{a}$

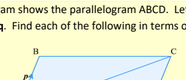
Important: Notice the negative sign for b since going in the opposite direction to the arrow in the diagram when we go from B to A

Now we have written $\overrightarrow{BC}$ in terms of lowercase letters and are done

Basic Examples

Example 1

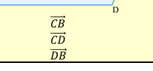
The diagram shows the parallelogram ABCD. Let $\overrightarrow{AB} = \boldsymbol{p}$ and $\overrightarrow{AC}$=**q**. Find each of the following in terms of **p** and/or **q**



i. $\overrightarrow{CB}$

ii. $\overrightarrow{CD}$

iii. $\overrightarrow{DB}$



Tip: opposite sides of a parallelogram are equal hence $\overrightarrow{AB} = \overrightarrow{DC} = \boldsymbol{p}$

i. $\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB} = -\boldsymbol{q} + \boldsymbol{p}$

ii. $\overrightarrow{CD} = -\overrightarrow{DC} = -\overrightarrow{AB} = -\boldsymbol{p}$

iii. $\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB} = \overrightarrow{AB} + \boldsymbol{p} - \boldsymbol{q}$

$= \boldsymbol{p} + \boldsymbol{p} - \boldsymbol{q}$

$= 2\boldsymbol{p} - \boldsymbol{q}$

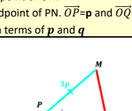
Alternative:

$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{CB} + \overrightarrow{AB} = \boldsymbol{p} - \boldsymbol{q} + \boldsymbol{p} = 2\boldsymbol{p} - \boldsymbol{q}$

Example 2

OMN is a triangle. P is the point on OM such that OP= $\frac{1}{4}OM$.
Q is the midpoint of ON.
i.R is the midpoint of PN. $\overrightarrow{OP}$=**p** and $\overrightarrow{OQ}$=**q**
ii.Find $\overrightarrow{QR}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$

i.



$\overrightarrow{QR} = \overrightarrow{QN} + \overrightarrow{NR}$

$= \boldsymbol{q} + \overrightarrow{NR}$

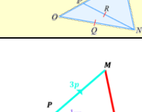
$= \boldsymbol{q} - \frac{1}{2}\overrightarrow{PN}$

$= \boldsymbol{q} - \frac{1}{2}\left(\overrightarrow{PO} + \overrightarrow{OQ} + \overrightarrow{QN}\right)$

$= \boldsymbol{q} - \frac{1}{2}(-p + q + q)$

$= \boldsymbol{q} - \frac{1}{2}(2q - p) = \boldsymbol{\frac{1}{2}p}$

ii.



$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OP} + \overrightarrow{PR}$

$= -q + p + \frac{1}{2}\overrightarrow{PN}$

$= -q + p + \frac{1}{2}\left(\overrightarrow{PO} + \overrightarrow{OQ} + \overrightarrow{QN}\right)$

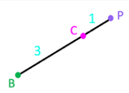
$= -q + p + \frac{1}{2}(-p + q + q)$

$= -q + p + \frac{1}{2}(2q - p)$

$= \boldsymbol{\frac{1}{2}p}$

Vectors With Ratio

We often have vectors given in a ratio. For example, $\overrightarrow{BC}:\overrightarrow{CP} = 3:1$

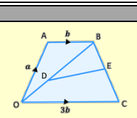


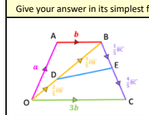
There are 2 ways to look at this:

Ratio Form 1: Write each piece as a part of the WHOLE LENGTH (more common)	Ratio Form 2: Write each piece in terms of the OTHER PIECE
Looking at BC $\overrightarrow{BC} = \frac{3}{4}$ of the **entire length** $\overrightarrow{BC} = \frac{3}{4}\overrightarrow{BP}$ **Looking at CP:** $\overrightarrow{CP} = \frac{1}{4}$ of the **entire length** $\overrightarrow{CP} = \frac{1}{4}\overrightarrow{BP}$ Notice how we have written $\overrightarrow{BC}$ and $\overrightarrow{CP}$ in terms of the entire length $\overrightarrow{BP}$ $\overrightarrow{BC} = \frac{3}{4}\overrightarrow{BP}$ and $\overrightarrow{CP} = \frac{1}{4}\overrightarrow{BP}$ Make sure you have realised that we use this way if we know or are able to find the whole/entire length.	Ratios are also just pushed over fractions!! $\frac{\overrightarrow{BC}}{\overrightarrow{CP}} = \frac{3}{1}$ Re-arranging gives $\overrightarrow{BC} = 3\overrightarrow{CP}$ This can be re-arranged to also $\overrightarrow{CP}$ make the subject $\overrightarrow{CP} = \frac{1}{3}\overrightarrow{BC}$ So, here we have a relationship between the pieces, not each piece as the whole line $\overrightarrow{BC} = 3\overrightarrow{CP}$ and $\overrightarrow{CP} = \frac{1}{3}\overrightarrow{BC}$ Make sure you have realised that we use this when we know only what one part of the entire length is and can't find the entire length.

Example of Ratio form 1

$OABC$ is a trapezium.
$\overrightarrow{OA} = \boldsymbol{a}$
$\overrightarrow{AB} = \boldsymbol{b}$
$\overrightarrow{OC} = 3\boldsymbol{b}$
D is the point on OB such that $OD:DB = 2:3$. E is the point on BC such that $BE:EC = 1:4$.
Work out the vector $\overrightarrow{DE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.
Give your answer in its simplest form.





The given ratios:

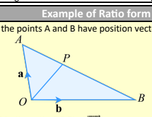
$OD:DB = 2:3 \Rightarrow \overrightarrow{OD} = \frac{2}{5}\overrightarrow{OB}$ and $\overrightarrow{DB} = \frac{3}{5}\overrightarrow{BC}$

$BE:EC = 1:4 \Rightarrow \overrightarrow{BE} = \frac{1}{5}\overrightarrow{BC}$ and $\overrightarrow{EC} = \frac{4}{5}\overrightarrow{BC}$

Way 1: Shorter Route	Way 2: Longer Route
$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BE}$	$\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BE}$
$\overrightarrow{DE} = \frac{3}{5}\overrightarrow{OB} + \frac{1}{5}\overrightarrow{BC}$	$\overrightarrow{DE} = -\frac{2}{5}\overrightarrow{OB} + \boldsymbol{a} + \boldsymbol{b} + \frac{1}{5}\overrightarrow{BC}$
$\overrightarrow{DE} = \frac{3}{5}(\boldsymbol{a} + \boldsymbol{b}) + \frac{1}{5}(-\boldsymbol{b} - \boldsymbol{a} + 3\boldsymbol{b})$	$\overrightarrow{DE} = -\frac{2}{5}(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{a} + \boldsymbol{b}$
Expand the brackets $\overrightarrow{DE} = \frac{3}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b} - \frac{1}{5}\boldsymbol{b} - \frac{1}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b}$	$+\frac{1}{5}(-\boldsymbol{b} - \boldsymbol{a} + 3\boldsymbol{b})$ Expand the brackets and collect like terms $\overrightarrow{DE} = -\frac{2}{5}\boldsymbol{a} - \frac{2}{5}\boldsymbol{b} + \boldsymbol{a} + \boldsymbol{b}$ $-\frac{1}{5}\boldsymbol{b} - \frac{1}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b}$
Collect like terms ($\boldsymbol{a}$ and $\boldsymbol{b}$) $\overrightarrow{DE} = \frac{2}{5}\boldsymbol{a} + \boldsymbol{b}$	Collect like terms ($\boldsymbol{a}$ and $\boldsymbol{b}$) $\overrightarrow{DE} = \frac{2}{5}\boldsymbol{a} + \boldsymbol{b}$

Example of Ratio form 1

In the diagram below the points A and B have position vectors a and b respectively.



The point P divides AB in the ratio 1:2.Find $\overrightarrow{OP}$

Let us now collect all the information we have found out:

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \boldsymbol{a} + \frac{1}{3}\overrightarrow{AB}$

$\overrightarrow{AB}$ means start at A and end at B. We need to write in terms of lowercase letters :

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\boldsymbol{a} + \boldsymbol{b}$

This allows us to write as

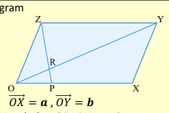
$\overrightarrow{OP} = \boldsymbol{a} + \frac{1}{3}(\boldsymbol{b} - \boldsymbol{a})$

Expand and simplify:

$= \boldsymbol{a} + \frac{1}{3}\boldsymbol{b} - \frac{1}{3}\boldsymbol{a} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

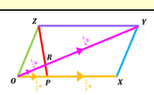
Example of Ratio form 1

OXYZ is a parallelogram



$\overrightarrow{OX} = \boldsymbol{a}$, $\overrightarrow{OY} = \boldsymbol{b}$

P is the point on OX such that OP : PX = 1 : 2
R is the point on OY such that OR : RY = 1 : 3
Work out in its simplest form, the ratio ZP : ZR



We can find the vectors directly.

$\overrightarrow{ZP} = \overrightarrow{ZO} + \overrightarrow{OP}$ doesn't work since don't know $\overrightarrow{ZO}$

$\overrightarrow{ZP} = \overrightarrow{ZY} + \overrightarrow{YX} + \overrightarrow{XP}$ doesn't work since don't know $\overrightarrow{YX}$

$\overrightarrow{ZP} = \overrightarrow{ZY} + \overrightarrow{YO} + \overrightarrow{OP}$ works

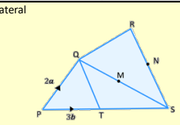
First, finding ZP	Next, find ZR
$\overrightarrow{ZP} = \overrightarrow{ZY} + \overrightarrow{YO} + \overrightarrow{OP}$ Since it is a parallelogram, ZY = OX hence $\overrightarrow{ZP} = \overrightarrow{OX} - b + \frac{1}{3}a$ $= a - b + \frac{1}{3}a$ $= \frac{4}{3}\boldsymbol{a} - \boldsymbol{b}$	$\overrightarrow{ZR} = \overrightarrow{ZY} + \overrightarrow{YR} = a - \frac{3}{4}b$

Hence, we find the ratio as

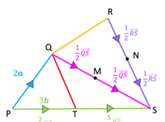
$ZP:ZR = \frac{4}{3}:1 = \mathbf{4:3}$

Example of Ratio form 2 (harder since less common)

PQRS is a quadrilateral



PTS is a straight line with PT : TS = 3 : 5
M is the midpoint of QS.
N is the midpoint of RS.
Write $\overrightarrow{MN}$ in term of **a** and **b**



If using form 1:

$\overrightarrow{PT} : \overrightarrow{TS} = 3:5$

$\overrightarrow{PT} = \frac{3}{8}\overrightarrow{PS}$

$\overrightarrow{TS} = \frac{5}{8}\overrightarrow{PS}$

If using form 2:

$\overrightarrow{PT} : \overrightarrow{TS} = 3:5$

$\frac{\overrightarrow{PT}}{\overrightarrow{TS}} = \frac{3}{5}$

$5\overrightarrow{PT} = 3\overrightarrow{TS}$

$\overrightarrow{PT} = \frac{3}{5}\overrightarrow{TS}$ and $\overrightarrow{TS} = \frac{5}{3}\overrightarrow{PT}$

$\overrightarrow{MN} = \overrightarrow{MS} + \overrightarrow{SN}$

$= \frac{1}{2}\overrightarrow{QS} - \frac{1}{2}\overrightarrow{RS}$

$= \frac{1}{2}\left(\overrightarrow{QP} + \overrightarrow{PT} + \overrightarrow{TS}\right) - \frac{1}{2}(2\overrightarrow{QT})$

$= \frac{1}{2}(-2a + 3b + \frac{5}{3}\overrightarrow{PS}) - (-2a + 3b)$

stuck here since don't know $\overrightarrow{PS}$

$= \frac{1}{2}\left(-2a + 3b + \frac{5}{3}(3b)\right) - (-2a + 3b)$

$= -\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} + \frac{5}{2}\boldsymbol{b} + 2\boldsymbol{a} - 3\boldsymbol{b} = \boldsymbol{a} + \boldsymbol{b}$

Tip: Opposite sides of a parallelogram are the same, so if you know the vector for one side you also know the vector for the other parallel side.

Parallel Vectors

Parallel means a **multiple** of (we can multiply one vector by another number to get the other vector)
For example: Consider the vectors $2\boldsymbol{a} - 3\boldsymbol{b}$ and $6\boldsymbol{a} - 9\boldsymbol{b}$. There are 3 way to prove these are parallel:

Way 1: Comparing by sight

We can see the first vector has been multiplied by 3 to get the second vector (or the second vector has been divided by 3 i.e. multiplied by $\frac{1}{3}$ to get the first vector

$6\boldsymbol{a} - 9\boldsymbol{b} = 3(2\boldsymbol{a} - 3\boldsymbol{b})$

$2\boldsymbol{a} - 3\boldsymbol{b} = \frac{1}{3}(6\boldsymbol{a} - 9\boldsymbol{b})$

Struggling to see what the number in green should be? There is a few nice things we can do (ways 2 and 3 below)

Way 2: Dividing like terms

Divide the coefficients of the common elements and you should get the same number if parallel

$2\boldsymbol{a} - 3\boldsymbol{b}$ and $6\boldsymbol{a} - 9\boldsymbol{b}$

Option 1: Work left to right	Option 2: Work right to left
$\frac{2}{6} = \frac{-3}{-9}$ $\frac{1}{3} = \frac{1}{3}$ The first vector is one third the second vector $2\boldsymbol{a} - 3\boldsymbol{b} = \frac{1}{3}(6\boldsymbol{a} - 9\boldsymbol{b})$ Hence the vectors are parallel (vectors are multiples)	$\frac{6}{2} = \frac{-9}{-3}$ $3 = 3$ The second vector is three times the first vector $6\boldsymbol{a} - 9\boldsymbol{b} = 3(2\boldsymbol{a} - 3\boldsymbol{b})$ Hence the vectors are parallel (vectors are multiples)

Way 3: Writing as a multiple and comparing coefficients

The following is another method but is not necessary here to do here as the example given above is easy (hence the method is a bit overkill).
However, you will see that this technique is an important technique in many areas of maths.
We first write the vectors as some unknown multiple (one vector is k times the other) and then compare like terms.

$2\boldsymbol{a} - 3\boldsymbol{b} = k(6\boldsymbol{a} - 9\boldsymbol{b})$

Expand and compare like terms (colour pairs)

$2\boldsymbol{a} - 3\boldsymbol{b} = 6k\boldsymbol{a} - 9k\boldsymbol{b}$

Option 1: Comparing coefficients of the $\boldsymbol{a}$ term	Option 2: Comparing coefficients of the $\boldsymbol{b}$ term
$2 = 6k$ $k = \frac{1}{3}$	$-3 = -9k$ $k = \frac{1}{3}$

We get the same value for k in each hence parallel

Two Basic Examples:

Are the vectors $2\boldsymbol{a} - 3\boldsymbol{b}$ and $-6\boldsymbol{a} + 9\boldsymbol{b}$ parallel?

$2\boldsymbol{a} - 3\boldsymbol{b}$ and $-6\boldsymbol{a} + 9\boldsymbol{b}$

Way 1	Way 2
$\frac{2}{-6} = \frac{-3}{9} = w$ $-\frac{1}{3} = -\frac{1}{3}$ The vectors are parallel since multiply the 2nd vector by $-\frac{1}{3}$ to get the 1st: $2\boldsymbol{a} - 3\boldsymbol{b} = -\frac{1}{3}(6\boldsymbol{a} - 9\boldsymbol{b})$	$\frac{-6}{2} = \frac{9}{-3}$ $-3 = -3$ These are parallel since multiply the 1st vectoe by -3 to get the 2nd vector: $6\boldsymbol{a} - 9\boldsymbol{b} = -3(2\boldsymbol{a} - 3\boldsymbol{b})$

Are the vectors $2\boldsymbol{a} - 3\boldsymbol{b}$ and $-6\boldsymbol{a} - 9\boldsymbol{b}$ parallel?

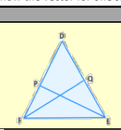
$2\boldsymbol{a} - 3\boldsymbol{b}$ and $-6\boldsymbol{a} - 9\boldsymbol{b}$

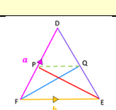
Way 1	Way 2
$\frac{2}{-6} = \frac{-3}{-9}$ $-\frac{1}{3} \neq \frac{1}{3}$ These vectors are **not parallel** since we multiply one part by $-\frac{1}{3}$ and other part by $\frac{1}{3}$	$\frac{-6}{2} = \frac{-9}{-3}$ $-3 \neq 3$ These vectors are **not parallel** since we multiply one part by -3 and the other part by 3

Tip: Opposite sides of a trapezium are parallel, so if you know the vector for one side you know the vector for the other parallel side is some multiple!

Example 1

DEF is a triangle.
P is the midpoint of FD.
Q is the midpoint of DE.
$\overrightarrow{FD}$ = a
$\overrightarrow{FE}$ = b
Use a vector method to prove that PQ is parallel to FE.





Let's find each vector $\overrightarrow{PQ}$ and $\overrightarrow{FE}$

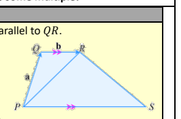
$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$ $= \frac{1}{2}\overrightarrow{FD} + \frac{1}{2}\overrightarrow{DE}$ $= \frac{1}{2}a + \frac{1}{2}(\overrightarrow{DF} + \overrightarrow{FE})$ $= \frac{1}{2}a + \frac{1}{2}(-a + b) = \frac{1}{2}\boldsymbol{b}$	$\overrightarrow{FE} = b$ (given in question)

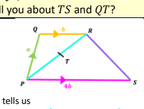
$\overrightarrow{PQ}$ is a scalar multiple of $\overrightarrow{FE}$

$\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{FE}$ hence parallel

Example 2

The diagram shows a trapezium $PQRS$. PS is parallel to QR.
$PS = 4QR$
$\overrightarrow{PQ} = a$ and $\overrightarrow{QR} = b$
The point T lies on the line PR such that $PT:TR = 4:1$.
i. Given that $\overrightarrow{TS} = k\overrightarrow{QT}$, find the value of k
ii. What does this tell you about TS and QT?





i.The ratio $\overrightarrow{PT} : \overrightarrow{TR} = 4:1$ tells us

$\overrightarrow{PT} = \frac{4}{5}\overrightarrow{PR}$ and $\overrightarrow{TR} = \frac{1}{5}\overrightarrow{PR}$

We need to find $\overrightarrow{TS}$ and $\overrightarrow{QT}$

$\overrightarrow{QT} = \overrightarrow{QP} + \overrightarrow{PT} = -a + \frac{4}{5}(a + b) = \frac{4}{5}b - \frac{1}{5}a$

$\overrightarrow{TS} = \overrightarrow{TP} + \overrightarrow{PS} = -\frac{4}{5}(a + b) + 4b = \frac{16}{5}b - \frac{4}{5}a = 4\left(\frac{4}{5}b - \frac{1}{5}a\right) = 4\overrightarrow{QT}$

$k = 4$

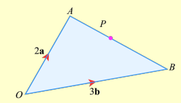
ii.

$\overrightarrow{TS} = 4\overrightarrow{QT}$

TS and QT are parallel since they are scalar multiples of each other.

Parallel Vectors Continued

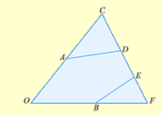
OAB is a triangle

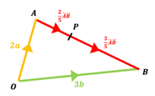


$\overrightarrow{OA} = 2\boldsymbol{a}$
$\overrightarrow{OB} = 3\boldsymbol{b}$
P is the point on AB such that AP:PB=2:3
Show that $\overrightarrow{OP}$ is parallel to the vector a+b

The diagram shows triangle OCF.
$\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = 3\boldsymbol{a}$, $\overrightarrow{OB} = 2\boldsymbol{b}$ and $\overrightarrow{OF} = 3\boldsymbol{b}$

Given that CD= $\frac{1}{3}CF$ and that CE= $\frac{5}{6}CF$
Use vectors to prove that AD and BE are parallel





$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$

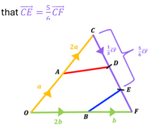
$= 3\boldsymbol{b} + \overrightarrow{BP}$

Remember our goal is to get rid of the big letters so we need to get rid of $\overrightarrow{BP}$ and write it in terms of small letters. We do this step by step.

Option 1:	Option 2:
$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$ $\overrightarrow{OP} = 3\boldsymbol{b} + \overrightarrow{BP}$ $= 3\boldsymbol{b} - \frac{3}{5}\overrightarrow{AB}$ $= 3\boldsymbol{b} - \frac{3}{5}(3\boldsymbol{b} - 2\boldsymbol{a})$ $= \frac{6}{5}\boldsymbol{a} + \frac{6}{5}\boldsymbol{b}$	$\overrightarrow{OP} = 3\boldsymbol{b} + \overrightarrow{BP}$ $= 3\boldsymbol{b} + \frac{3}{5}\overrightarrow{BA}$ $= 3\boldsymbol{b} + \frac{3}{5}(2\boldsymbol{a} - 3\boldsymbol{b})$ $= \frac{6}{5}\boldsymbol{a} + \frac{6}{5}\boldsymbol{b}$

i

Given that $\overrightarrow{CD} = \frac{1}{3}\overrightarrow{CF}$ and that $\overrightarrow{CE} = \frac{5}{6}\overrightarrow{CF}$



$\overrightarrow{CD} = \frac{1}{3}\overrightarrow{CF} = \frac{1}{3}(-3a + 3b) = b - a$

$\overrightarrow{CE} = \frac{5}{6}\overrightarrow{CF} = \frac{5}{6}(-3a + 3b) = \frac{5}{2}b - \frac{5}{2}$

We need to find $\overrightarrow{AD}$ and $\overrightarrow{BE}$

$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$
$= 2a + b - a$
$= \boldsymbol{a} + \boldsymbol{b}$

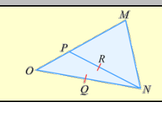
$\overrightarrow{BE} = \overrightarrow{BF} + \overrightarrow{FE}$

$= b + \overrightarrow{FE}$

$= b - \frac{1}{6}\overrightarrow{CF} = b - \frac{1}{6}(-3a + 3b)$

$= b - \left(-\frac{1}{2}a + \frac{1}{2}b\right) = b + \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} = \frac{1}{2}\overrightarrow{AD}$

OMN is a triangle. P is the point on OM such that OP= $\frac{1}{4}OM$. Q is the midpoint of ON. R is the midpoint of PN.
$\overrightarrow{OP}$=p, $\overrightarrow{OQ}$=q



Find in terms of p and q
i. $\overrightarrow{MN}$
ii. $\overrightarrow{PR}$

i.
We want to find $\overrightarrow{MN}$ which starts at M and ends at N
$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$
We can find $\overrightarrow{MO}$ and $\overrightarrow{ON}$
Given $\overrightarrow{OP} = \frac{1}{4}\overrightarrow{OM}$. We can re-arrange to find $\overrightarrow{MO}$
$\overrightarrow{OM} = 4\overrightarrow{OP}$
$\overrightarrow{MO} = -4\overrightarrow{OP} = -4\boldsymbol{p}$
Since Q is the midpoint of ON, $\overrightarrow{ON} = 2\overrightarrow{OQ} = 2q$
Next, we substitute our values in the equation $\overrightarrow{MN}$
$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON} = -4p + 2q$

ii.
We want to find $\overrightarrow{PR}$ which starts at P and ends at R. Since R is the midpoint of PN, $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{PN}$
Now, we find PN
$\overrightarrow{PN} = \overrightarrow{PO} + \overrightarrow{ON} = -p + 2q$
$\overrightarrow{PR} = \frac{1}{2}(-p + 2q) = -\frac{1}{2}p + q$

For QR to be parallel to OP, it must be a scalar factor of it. Let us find QR first.

$\overrightarrow{QR} = \overrightarrow{QN} + \overrightarrow{NR}$

$= q - \frac{1}{2}(-p + 2q)$

$= \frac{1}{2}p = \frac{1}{2}\overrightarrow{OP}$

Since we have shown QR to be ½ times OP, they must be parallel.

Vectors – Proving Straight Lines

For example, the straight line ABC

We are often asked to prove something is a straight line, so do this we **find any two of the 3 pieces on a line and prove parallel** (recall that when we have a straight line each piece of the line can be written as a multiple (meaning parallel). For our example above, the 3 pieces on the line are**.**

All we have to do is prove either one of the following pairs **are parallel**:

Pair 1	Pair 2	Pair 3
Notice how parallel didn't guarantee they were the same line. We have to also mention that both pieces **share the same point** since this makes them the same line.		
The lines both share point B Therefore, ABC must be a straight line 	The lines both share point C Therefore, ABC must be a straight line 	The lines both share point A Therefore, ABC must be a straight line

So, to **prove a line is a straight line**, you just have to prove

- Any 2 pieces of the line are parallel
- These pieces share a common point

Note: This also works the other way. When we are told something is a straight line, we can choose any two pieces on it and use the fact that they are parallel!

In the diagram, $\overrightarrow{OA} = 4\boldsymbol{a}$ and $\overrightarrow{OB} = 4\boldsymbol{b}$. OAC, OBX and BQC are all straight lines. $AC = 2OA$ and $BQ:QC = 1:3$.
Given that $\overrightarrow{BX} = 8\boldsymbol